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DYNAMIC ANALYSIS OF FALL OF A HIGH BUILDING

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Abstract: The differential equation of collapse of a high building is derived taking into account many influences. Computer simulation of the collapse of the WTC building is presented using two independent programs for some variations of parameters. The results of both, differential equation and computer simulation, are compared.

Keywords: structural mechanics, dynamics, collapse

1 Introduction

This article deals with fall of a high building. Its aim is not to investigate a cause of the fall but to examine the falling process itself. The article studies theory of fall of a high building. Process of the falling is investigated from the point of view of basic laws of mechanics. Differential equation of a high building collapse is derived and all major influences of the falling process are included. Results of the computer simulation are presented.

2 Derivation of a differential equation of fall of a high building

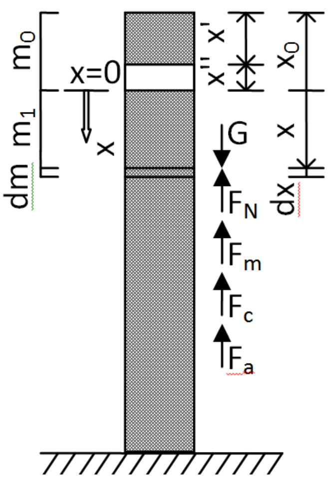

Fig. 1. The scheme of falling building

Let's assume that columns in the location between the coordinates x' and x_0 lose stability and a top part of the building above x' starts to fall and hits the still undamaged lower part of the building under the location x_0 with velocity v_0.

We will introduce equation of dynamical equilibrium for the location x:

$$G - F_N - F_m - F_C - F_a = 0 \qquad (1.1)$$

G is weight of a part of the building above the location x, for which equilibrium equation is formulated

F_N is resistance put up by the columns against the collapse

F_m is resistance originated by hitting of a falling part of the building into a motionless mass

F_C is a viscous damping

F_a is an inertial force of a falling mass

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2.1 Derivation of individual parts of the equilibrium equation (1.1)

a) The weight of the building above the location x:

$$G = mg\beta \text{, where} \qquad (1.2)$$

m is the mass of the building above the location x, g is acceleration of gravity, β is portion of the total mass above the location x which pushes to a lower part of the building. Mass which falls outside of the building is subtracted.

b) The columns resistance:

$$F_N = mgs\kappa \qquad (1.3)$$

s is a rate of the ultimate force of columns to the current force in columns in the moment of the collapse, κ is the factor of the ultimate force of columns which represents average column resistance during its deformation related to an ultimate force.

Computation of column pressing was done using the method of controlled deformation in order to investigate this factor and receive its operational chart (see picture below). Factor κ is then the rate of the ultimate force to its median value.

c) The resistance of a motionless mass:

It is inertial force of a still mass dm accelerated in a time *dt* to the speed v ($a = dv/dt$). We can use the term $a = v/dt$ for acceleration in the equation due to acceleration starting from zero up to the speed *v*.

The force F_m can be then expressed in this way:

$$F_m = dm \cdot a = dm \cdot \frac{v}{dt} \qquad (1.4)$$

When considering that $v = dx/dt$, the equation (1.4) can be rewritten as follows:

$$F_m = dm \cdot \frac{v^2}{dx} = \mu v^2 \text{, where} \qquad (1.5)$$

$\mu = dm/dx$ is a line density of the building.

d) The viscous damping:

$$F_c = C \cdot v = m\alpha v \text{, where} \qquad (1.6)$$

C is a factor of the viscous damping. We are considering Rayleigh damping here which is depending on mass quantity $C = m\alpha$ only.

e) The inertial force of a falling mass:

$$F_a = \beta m \cdot a = \beta m \frac{dv}{dt} = \beta m v \frac{dv}{dx} \qquad (1.7)$$

Again, only the inertial force of a mass which does not fall outside of the building is considered here.

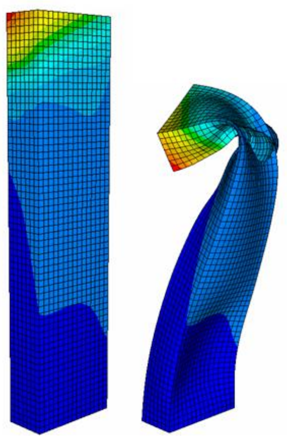

Fig. 2. Undeformed and deformed column

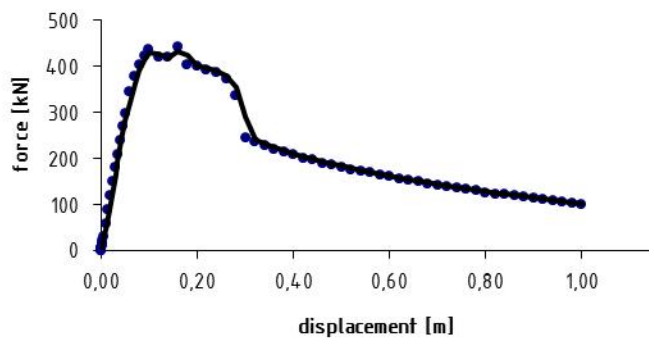

Fig. 3. Response diagram of pressing of the column

3 Differential equation of the building collapse

By substituting relations which we derived above into the equation (1.1) we get:

$$mg\beta - mgs\kappa - \mu v^2 - m\alpha v - \beta m v \frac{dv}{dx} = 0 \qquad (1.8)$$

We will divide the equation with speed v and mass m and adjust:

$$\frac{b}{v} - \frac{v}{x + x_0} - \alpha - \frac{\beta dv}{dx} = 0 \text{, where} \qquad (1.9)$$

$b = g(\beta - s\kappa)$. We used relation $\mu(x + x_0) = m$ when adjusting the equation.

Analytical solution of the differential equation was not found so we are going to solve the equation (1.9) with numerical method. The Euler implicit method of solving a differential equation was found to be the most suitable method. Its principle is:

$$v_{i+1} = v_i + h \cdot f(x_i, v(x_i)) \qquad (1.10)$$

We will get this in our equation:

$$v(x_{i+1}) = v(x_i) + \frac{h}{\beta}\left(\frac{b}{v(x_{i+1})} - \frac{v(x_{i+1})}{x_{i+1} + x_0} - \alpha \right) \qquad (1.11)$$

After deduction of the speed $v(x_{i+1})$ we get this relation:

$$v(x_{i+1}) = \frac{1}{2(h + \beta x_{i+1} + \beta x_0)} * \\ * \left\{ \begin{array}{l} -\alpha h x_{i+1} + \beta v(x_i) x_{i+1} - \alpha h x_0 + \beta v(x_i) x_0 + \\ + \sqrt{-4(h + \beta x_{i+1} + \beta x_0)(-bhx_{i+1} - bhx_0) + (\alpha h x_{i+1} - \beta v(x_i) x_{i+1} + \alpha h x_0 - \beta v(x_i) x_0)^2} \end{array} \right\} \qquad (1.12)$$

We have to compute the speed $v(0)$. We will start from the same differential equation where we will modify factor b into $b_0 = (\beta - s_0\kappa)g$, where $s_0 < 1$:

$$\frac{b_0}{v(x)} - \frac{v(x)}{x + x'} - \frac{\beta dv(x)}{dx} = 0 \qquad (1.13)$$

The solution will thus have a similar form. We will use boundary conditions $v(0) = 0$. Then we are looking for $v(x'') = v_0(x)$ which is our first v_{x_i} from (1.12).

3.1 Discussion of the extend of damping, the safety factor and the ultimate force ratio

Before we start solving equation in the numerical way we shall clarify the magnitude of the damping α.

$$\alpha = \frac{C}{m} = \frac{2m\omega_n\xi}{m} = 2\omega_n\xi \qquad (1.14)$$

For the ratio of damping, we will consider the value 10-30% and the limiting value 0 that yields the damping ratios $\alpha = 0{,}147, \alpha = 3{,}18, \alpha = 0$. We will consider value 0 in order to solve the collapse without the effect of damping.

Much less uncertainty is about the value of the parameter *s*. We are assuming it to be around 2.5 – 3.

The coefficient of the ultimate strength in columns κ was computed from simulation of pressing of the columns by the method of controlled deformation.

It is clear from the graph that the value of the κ coefficient will be around 0.25.

3.2 Computer simulation

For computer simulation of fall of a high building the RFEM and FyDiK programs were used. Both programs used the dynamic relaxation method. The RFEM is a finite element program whereas the FyDiK uses an inverse approach, mass points connected by elastoplastic

Fig. 4. Deformed building from RFEM (α=0.5;s=3)

Fig. 5. Deformed building from FyDiK (α=0.5;s=3)

springs. The differencies between the results of both programs was small. In the figures 1. and 2. we can see resulting deformation of the building after the fall has stopped. For the chosen parameters, which the authors believe could be probable, the fall of the building would stop after falling cca 80 m.

3.3 Results – times and extends of the fall for various parameters

Table 1. Times and extends of the fall for various parameters, results of solution of differential equation and computer simulation by the RFEM and FyDiK programs

s[-]	α	The theory without falling away of a mass x[m]	The theory with falling away of a mass x[m]	RFEM x[m]	FyDiK x[m]	The theory without falling away of a mass t[s]	The theory with falling away of a mass t[s]	RFEM t[s]	FyDiK t[s]
2.0	0.147	330	330	331.0	278.7	15.2	25.8	15.7	19.0
2.0	3.18	330	330	18.2	63.2	83.1	357.1	7.4	31.5
3.0	0	330	330	259.8	287.1	20.5	36.1	17.9	20.4
3.0	0.147	330	330	324.5	304.2	24.2	103.0	28.6	33.1
3.0	0.5	330	330	79.6	72.3	36.5	330.5	12.1	28.1
3.0	1.06	330	330	64.6	66.5	60.2	707.3	15.9	37.4
2.0	0.147	330	330	331.0	278.7	15.2	25.8	15.7	19.0
2.0	3.18	330	330	18.2	63.2	83.1	357.1	7.4	31.5

4 Conclusion

The differential equation of fall of a high building was derived and solved by the Euler method. Two independent computer programs, the RFEM and the FyDiK, were used for the simulation of the fall. Despite the difference in the approaches both computer programs gave comparatively similar results. The difference between the result of the differential equation and that of the computer simulation is greater. The computer simulation gives more reliable solution as there is no need to keep continuity of all values used in differential equation so the approximation to the reality is better.

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